Relevance as MacGuffin in Mathematics Education[1]

Dylan Wiliam
King's College London School of Education

0 Abstract

Alfred Hitchcock's metaphor of a MacGuffin—a plot device primarily intended to motivate the action in a film, and to which relatively little attention is paid—is used to illustrate the use of 'realistic' contexts in mathematics education. Contexts used for mathematics teaching are classified into three kinds:

contexts which bear little or no relation to the mathematics being taught, and which serve primarily to legitimate the subject matter ('maths looking for somewhere to happen');

contexts having an inherent structure with elements that can be mapped onto the mathematical structures being taught ('realistic mathematics');

contexts in which the primary aim is the resolution of a problem in which no particular (or even any!) mathematics need necessarily be used ('real problems').

Since examples of the last of these three are comparatively rare, and of the first are common, but difficult to justify, attention is focused on the second, and examples from Dutch and British curricula are discussed. 'Realistic' contexts are characterised by the extent to which the contexts are shared by students, and their fit with the mathematical structures being taught, and suggestions for strategies for choosing such contexts are made.

1 Introduction

This paper explores the use of contexts and metaphors in the presentation of mathematical texts. For an aspect of mathematics education that has such a profound and widespread influence on the practice of teachers, and on the learning of students, the role of text and textbooks in mathematics education has been the subject of comparatively little analysis. Even such analysis as there is has been firmly rooted in the world of pure mathematics, where new mathematical situations are presented in terms of situations arising in the mathematical world (Otte, 1986; Van Dormolen, 1986). However, even the briefest observation of the practice of teachers will show that, even where contexts and metaphors are not present in the textual materials that teachers use, contexts and metaphors are introduced by teachers in order to make the mathematical ideas more meaningful and more relevant to their students. The introduction of situations from the world outside the mathematics classroom has blurred the distinction between pure mathematics and applied mathematics. In such examples, the real world is not the source of mathematics, nor even the reference point for it. Rather, the real world is a source of analogies that can be used to capitalise on existing mental schemas that learners bring with them to the mathematics classroom. For example, negative numbers are commonly introduced via games of gain and loss, the height of land above or below some datum (ground floor, sea level) or temperatures above and below sea level (Freudenthal, 1983 p436).

The use of such contexts has nothing to do with the problems of measuring temperatures below zero or of measuring the height of land below sea level—in other words, the use of such contexts has nothing to do with applied or applicable mathematics. At the risk of self-reference, I will explore the use of contexts and metaphor in the presentation of mathematical ideas by the use of a metaphor—the idea of a MacGuffin, as used by the film director Alfred Hitchcock. In defence of the heaping of metaphor on metaphor, I note that Richards was quite robust in the defence of this practice, as, in effect, being inevitable (Richards, 1979 p101) and pointing out the difficulty that one gets into if one tries to avoid doing so.

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The intention is to clarify the distinction between the use of contexts as metaphors for aspects of pure mathematics on the one hand and contexts as sources of mathematical activity on the other.

2 Hitchcock's definition of MacGuffin

In many of his films. Alfred Hitchcock makes use of an idea which he calls a MacGuffin. This is essentially little more than a theatrical device that motivates the various actors in the drama.

> Hitchcock That ... was our MacGuffin. I must tell you what that means.
>
> Truffaut Isn't the MacGuffin the pretext for the plot?
>
> Hitchcock Well, it's the device, the gimmick, if you will, or the paper that the spies are after. I'll tell you about it. Most of Kipling's stories, as you know, were set in India, and they dealt with the fighting between the natives and the British on the Afghanistan border. Many of them were spy stories, and they were concerned with the efforts to steal the secret plans out of a fortress. The theft of secret document was the original MacGuffin. So the 'MacGuffin' is the term that we use to cover all that sort of thing: to steal plans or documents or discover a secret. It doesn't matter what it is. And the logicians are wrong in trying to figure out the truth of a MacGuffin, since it's beside the point. The only thing that really matters is that in the picture the plans, documents, or secrets must seem of vital importance to the characters. To me, the narrator, they're of no importance whatsoever.
>
> You may be wondering where the term originated. It might be a Scottish name, taken from a story about two men in a train. One man says, "What's that package up there in the baggage rack?"
>
> And the other answers, "Oh that's a MacGuffin." The first one asks, "What's a MacGuffin?"
>
> "Well," the other man says, "it's an apparatus for trapping lions in the Scottish Highlands."
>
> The first man says, "But there are no lions in the Scottish Highlands," and the other one answers, "Well then, that's no MacGuffin!" So you see that a MacGuffin is actually nothing at all (Truffaut, 1978 pp157-158).

Another important feature of the MacGuffin is that the details of the MacGuffin are of secondary importance, and often the planning of them is left until quite late in the devising of the plot, as this extract from the same interview illustrates:

> Hitchcock When I started to work with Ben Hecht on the screenplay for *Notorious*, we were looking for a MacGuffin, and as always, we proceeded by trial and error, going off in several directions that turned out to be too complex. The basic concept of the story was already on hand (Truffaut, 1978 p197).

In some cases, hardly any attention is paid to the MacGuffin at all.

> Hitchcock My best MacGuffin, and by that I mean the emptiest, the most non-existent, and the most absurd, is the one we used in *North by Northwest*. The picture is about espionage, and the only question that's raised in the story is to find out what the spies are after. Well, during the scene at the Chicago airport, the Central Intelligence man explains the whole situation to Cary Grant, and Grant, referring to the James Mason character, asks, "What does he do?"
>
> The counter-intelligence man replies, "Let's just say that he's an importer and exporter."
>
> "But what does he sell?'
>
> "Oh, just government secrets!" is the answer.
>
> Here, you see, the MacGuffin has been boiled down to its purest expression: nothing at all! (Truffaut, 1978 p160)

It is my contention that the MacGuffin provides an appropriate metaphor for understanding the role of the relevance of mathematical concepts as implemented in many mathematics curricula.

3 'Realistic' mathematics?

Almost all published schemes for the teaching of mathematics use contexts from outside mathematics itself in order to communicate mathematical ideas. Below are some examples of tasks that attempt to use 'real-life' contexts in order to make mathematical tasks more relevant to learners.

One example of a task embedded in a context from which it is relatively independent is the game of Matrix Soccer included in one of the packs developed by the Mathematics for the Majority Continuation Project —a project specifically aimed at provide learning materials for non-academic pupils forced to remain at school by the raising of the school leaving age to 16.

This game is essentially a drill and practice exercise to encourage students to remember that when a 2-dimensional column vector is used to represent displacement in the plane, the upper number is the displacement to the left (negative) or right (positive), and the lower number is the displacement up or down, although in the game, the lower numbers are all positive.

Each player in the game has cards that display a vector, and players take turns in moving the ball according to the vector shown on the card. Since the cards are played at random, very little skill is involved, and goals are quite rare.

There are no features of the structure of the game of soccer, apart from the use of a ball and a goal, that would indicate that soccer should be used as the context for the task. It would be equally appropriate to choose lacrosse or hockey, or to make the task an abstract 'board' game. However, this is not the point. In this example, the primary purpose of the context is to convince the student that the task that the teacher is presenting to the pupil is a worthwhile activity, by virtue of being the kind of activity in which people not studying in mathematical classrooms engage—an example of 'mathematics looking for somewhere to happen'. The context is, essentially, a MacGuffin.

Another example comes from a mathematics textbook aimed at 'low attainers' in the final years of compulsory education:

> [Next to a picture of a middle-aged man drinking a pint of beer]
>
> Alan drank $\frac{5}{8}$, of his pint of beer.
>
> What fraction was left ? (Harper, 1984 p105)

Apart from cueing the need for subtraction, the context of this task is completely independent of the mathematics intended. Beer is not measured in eighths of pints, and even were it to be so measured, it is unlikely that Alan would be thinking about the measurement while drinking. We have a mathematical task, which is $1 - \frac{5}{8}$, set in a particular context, but there is nothing about this particular context that might help the student identify an appropriate strategy. Again, the context here is, essentially, a MacGuffin—the mathematics is 'looking for somewhere to happen'. The choice of this particular context, included as it is in a book entitled 'Mathematics at work', seems to be used solely to convince disaffected low-attaining fifteen- and sixteen-year-olds that this is 'real' mathematics, done by 'real' people. In a very real sense, the situation is a 'con'-text—a deception that the activity is worthwhile.

In this, admittedly extreme, example, the only purpose of the context is to cue the students to the mathematical operation to be used, although this is likely to be trivially easy. After all, there can be very few students who could perform the subtraction operation while being unable to identify subtraction as the required operation. After all, this is an activity at which students get much practice. The extent to which 'spotting the operation' is part of the 'community of practice' (Lave & Wenger, 1991) of mathematics classrooms is shown by the following problem.

> There are 26 sheep and 10 goats on a ship.
> How old is the captain?

When this item was given to a sample of 90 first grade students in the United States, three-quarters answered 36 (Schoenfeld, 1990)!

Now the stereotyped nature of such items has been criticised for many years (Nesher, 1980), but despite the widespread consensus on their poverty, such problems form a considerable proportion bulk of the problems encountered by students even in supposedly 'innovative' curricula.

‘Realistic Mathematics Education’ (Streefland, 1991) purports to be an improvement on previous curricula due to its emphasis on ‘realistic’ interpretations of problem-situations. The paradigm is exemplified by items such as the following taken from the 1992 pilot national curriculum mathematics tests for 14-year-olds, and cited by Cooper (1992).

> This is the sign in a lift at an office block:
>
> This lift can carry up to 14 people
>
> In the morning rush, 269 people want to go up in the this lift.
> How many times must it go up?

Since the test is intended for 14-year-olds, and since they are allowed access to calculators, most students had little difficulty in realising that the solution to the item involved dividing 269 by 14. On a typical 10-digit calculator, this yields the answer 19.21428571. Responses which involve merely write down this answer, or any other non-integral answer, are regarded as ‘non-realistic’—only 20 is regarded as a ‘realistic’ response. However, as Cooper points out, arriving at the ‘correct’ answer of 20 requires many assumptions (eg that the lift is full apart from the last trip, that no-one gets fed up and uses the stairs, that the restriction is based strictly on number rather than mass or volume, so that for example, none of the people using the lift are wheelchair users).

In this sense, so-called ‘realistic’ problems involve almost as many stereotypical assumptions as the ‘stereotyped’ examples over which they are supposed to be an improvement, but it might be argued that the context does cue students into appropriate strategies. For example, even if the student cannot represent this as a division problem, the sequential nature of the lift journeys does suggest a model of repeated subtraction. Moreover, the interim stages in the repeated subtraction can be mapped directly onto the ‘real world’ situation (eg the diminuend as the number of people still waiting to use the lift).

However, it is often the case that so little thought is given to the context that it is not only irrelevant to the task, it can actually prevent the learner from reaching the ‘correct’ answer (ie the answer required by the teacher).

> [Next to a picture of a disc-jockey at work]
>
> A disc jockey plays two records. The first lasts $2\frac{5}{8}$ minutes and the second lasts $3\frac{1}{4}$ minutes. Find the total time taken to play both records (Lawson, 1984 p17).

Now in the ‘real world’ (ie the world outside the mathematics classroom, or, even better, outside the school), no one measures time in eighths of minutes. Furthermore, in the ‘real world’ disc-jockeys almost invariably talk between records, or play jingles, or miss out instrumental introductions. Because of this, the answer “around 6 minutes” might be regarded as *more* appropriate (realistic?) than the ‘correct’ answer of $5\frac{7}{8}$ minutes.

Another such example is the task ‘Logical Kitty’ (SMILE, 1990a), presented in the form of a comic strip, which uses the situation of a young woman getting ready to go out to a disco. She has three pairs of each of three different kinds of socks in a drawer, and is about to select a pair of socks to wear to the disco when there is a power failure. The task then asks “How many socks does Kitty need to take from the drawer to be sure of getting a matching pair?” The following resolution of the task was given by a twelve-year-old student, in the teacher’s journal *Splash*.

> Dear Splash
>
> I think Kitty will need to take only two socks because if Kitty is going to a disco it doesn’t really matter because no one is going to care about her socks. Just her dancing.
>
> by Louise Brooks, 8O, Morpeth School (Brooks, 1993 p23)

The fact that this response is by a female student is not, I think, irrelevant. Over the last twenty years, considerable evidence has been accumulated that females and males differ in their preferred ‘ways of knowing’ (Belenky, Clinchy, Goldberger, & Tarule, 1986).

Much of this work was precipitated by the work of Lawrence Kohlberg who explored stages of moral development through analysis of students’ responses to ‘dilemmas’.

> In Europe, a woman was near death from a rare form of cancer. There was one drug that the doctors thought might save her, a form of radium that a druggist in the same town had recently discovered. The druggist was charging $2000, ten times what the drug cost him to make. The sick woman’s husband, Heinz, went to everyone he knew to borrow the money, but he could only get together about half what the drug cost. He told the druggist that his wife was dying and asked him to sell it cheaper, or let him pay later. But the druggist said, “no.” So Heinz got desperate and broke into the man’s store to steal the drug for his wife. (Kohlberg, 1976 p42)

By analysing the responses of students to the question “Should Heinz have stolen the drug?”, Kohlberg derived a hierarchy of moral growth, based on the reasons given to support the answer to the question posed, ranging from answers based on principles of punishment and reward to those based on abstract principles of justice.

The development of the hierarchy was based entirely on the responses of young males. When Kohlberg found that the responses of females did not fit into the structure he had developed, he “concluded that their deviation from the established hierarchical scheme implied an arrested form of moral development” (Brown, 1984 p11).

One of Kohlberg’s associates, Carol Gilligan, pointed out that the prevalent response of males is to accept the dilemma, and try to work within the problem structure given, whereas females tend not to accept the dichotomy presented by the ‘dilemma’. The following response by Amy is typical.

> Well, I don’t think so. I think there might be better ways besides stealing it, like if he could borrow the money or make a loan or something, but he really shouldn’t steal the drug—but his wife shouldn’t die either. If he stole the drug, he might save his wife then, but if he did, he might have to go to jail, and then his wife might get sicker again, and he couldn’t get any more of the drug, and it might not be good. So they really should just talk it out, and find some other way to make the money. (Gilligan, 1982 p28)

It is not that Amy does not understand the dilemma or the moral issues involved. Rather she has taken the dilemma on as a *real* problem, embedded in the world of social beings, rather than an abstract logical puzzle.

Gilligan termed the kind of de-contextualised, abstract, de-personalised thinking exemplified by the boys’ responses as ‘separate’ while girls tended to respond in a way that she termed ‘connected’, characterised by an awareness of context, and a concern with relationships between people.

Thus when they are presented with a ‘real-world’ problem, many girls seek to relate the problem to their existing knowledge, supplying any information that they feel is missing from their own experience, while boys are often content to tackle the problem in isolation from their previous experience. Boaler (1994) found that the use of purportedly ‘girl-friendly’ contexts such as ‘fashion’ actually prevented girls from demonstrate mathematical achievement that they did display in relatively de-contextualised items, while their performance in ‘boy-friendly’ contexts such as football was much more closely related to their performance on the abstract tasks.

The use of ‘real-world’ contexts can also lead some students to challenge the assumptions on which the problem is based—what Stephen Brown (1984) calls ‘de-posing’ the problem. In the development of assessment materials for the national assessments of 14-year-olds in design and technology, students were asked to explore themes such as ‘Hypothermia’. In one school where this activity was piloted, almost all the boys began by investigating the thermal insulation properties of different materials. However, several groups of girls questioned the assumptions behind the task, asking why it was that so many old people could not afford to heat their homes adequately in winter (Consortium for Assessment and Testing in Schools Key Stage 3 Technology Team, 1991). As a solution to the real world problem, such a

‘lateral’ approach to a complex problem is highly valued in the world of work. In the context of school assessments, it is likely to lead to zero marks for ‘not reading the question’.

The use of contexts can therefore be regarded as having two independent dimensions. The first is the use of contexts to motivate students by the appearance of *relevance* to the real world or to life outside the mathematics classroom.

In a few very rare classrooms—for example those of Phoenix Park School described by Jo Boaler (1997)—students learn that their experience of the world outside the mathematics classroom *is* relevant to the activities they undertake in the mathematics classroom, and, *as a direct result*, are able to use the knowledge gained inside the mathematics classroom when solving problems elsewhere.

However, in the vast majority of classrooms, relevance is a MacGuffin—a device to motivate learners; to convince them that the activities they are given are some how *of* the real world, even though they do not appear to be connected *to* it. Students know this, and as a result their thinking in mathematics lessons becomes divorced even from their thinking elsewhere in school, let alone the world outside school. Mathematics lessons thus become literally mindless—an activity in which students come to believe that thinking is not helpful (Boaler, 1997).

The second dimension of the use of ‘real-world’ contexts is based not on their relevance, but from the fact that metaphorical relationships can be set up between structures in the ‘real world’ and the mathematical structures to be taught. However, the choice of these contexts, and the way that the metaphorical relationships are set up is of crucial importance to their value, and in the last two sections of this paper, examples of the use of such contexts are discussed, and some guidelines for the choice of such contexts are made.

4 The use of ‘realistic’ contexts in mathematics materials

A more sophisticated use of context is exemplified by the approach to the addition and scalar multiplication of matrices employed by the School Mathematics Project in England (School Mathematics Project, 1968) and the Dutch Realistic Mathematics project (Lange Jzn, 1990).

In both these presentations, matrices are defined as ways of storing information about items that are classified in two ways:

In the case of Realistic Mathematics—size and make of jeans;
In the case of SMP—house number and variety of milk bottles.

Addition of matrices is then defined in terms of aggregating quantities from consecutive days or weeks. However, when the topic of scalar multiplication of matrices is tackled, and in particular, the multiplication of a matrix by a column vector, there is a difference in the approaches adopted. The Realistic Mathematics materials introduce the idea of a scalar product by giving the profit on each make of jean (Lange Jzn, 1990 p111). This has the effect of introducing some relatively large numbers into the arithmetic, and it may have been a concern to limit the size of the numbers in the complicated arithmetic of scalar multiplication that prompted the SMP authors to dispense with the milk delivery context once the idea of matrix addition had been established. In the SMP treatment, the context used for the introduction of multiplication of a matrix by a column vector is that of an athletics competition between three schools. The entries in the matrix are the numbers of 1st, 2nd, 3rd and 4th places gained by each school. The plausibility of the arithmetical procedure for multiplying a matrix by a column vector is established by the need to discover which school won overall, given that the number of points awarded for each 1st, 2nd, 3rd and 4th place is 5, 3, 2, and 1 respectively.

In order to explain the procedure for multiplying two non-columnar matrices together, the existence of rival scoring models is hypothesised—in this case 8, 5, 3 and 1 points respectively for each 1st, 2nd, 3rd and 4th place, which (of course!) has the effect of changing the rank ordering of the three schools in the athletics match. At this point, the use of the context is becoming fairly strained in that while the structure in the ‘real’ situation does have parallels with the mathematical structure of matrices, the situation that has been

engineered is beginning to look very contrived. However, the contrived nature of the situation is probably more a reflection of the 'unnatural' nature of the process of multiplying matrices, than any deficiency of the particular context chosen by SMP. It seems likely that any extension of the jeans shop context to describe multiplication by non-columnar matrices would appear to be equally contrived.

The distinction between the two approaches can also be examined in terms of the *parsimony* of the contexts used, and the use of *abstract* examples.

The Realistic Mathematics approach stays firmly within the single context established—all the examples flow as if from the same 'case-study'. In contrast, the SMP treatment, having established the plausibility of the nature of the operation being presented immediately moves on to examples of abstract decontextualised matrices of various shapes.

The context used by SMP is plausible, but is not designed to be particularly relevant to the students for whom the material is designed: it seems unlikely that the majority of such students aspire to deliver milk for a living. The Realistic Mathematics approach, on the other hand, by identifying particular brands of jeans (Wrangler, Levis, etc) seems to aim more for a direct motivation by virtue of the context, another example of relevance as MacGuffin.

However, the primary use of the context here is not motivational, but structural. The *relevance* of the context is almost completely incidental to the choice of context. The most important thing about the contexts chosen is that aspects of the structure of the real situation can be represented by the mathematical structure of matrices. In other words they are *metaphors* for the mathematical structure of matrices. Contexts are employed in curricular materials to capitalise on *scripts* (Schank & Abelson, 1977), *schemas* or *frames* (Minsky, 1975) that learners already have in order to produce responses from the learner that converge on the desired mathematical responses.

5 Criteria for the selection of models

Many different contexts are used in the presentation of introductory work on negative numbers or directed magnitudes. One example presented by Freudenthal (1983) is the use of counters of two colours (in this case black and red) with the rule that a pair of one red counter and one black counter annihilate each other. The fact that 7 black and 3 red counters in, in some sense, 'the same' as 6 black and 2 red counters leads naturally to the notion of integers as equivalence classes of pairs of integers.

However, the contexts used in other curriculum schemes are much more varied. Temperatures above and below zero, heights above and below sea-level, and dates before and after the birth of Christ are all commonly found, and, of course, there are many other more idiosyncratic approaches.

When we come to compare these approaches, then three aspects of the context or model seem to be particularly important:

commonality
match
range

Each of these is discussed in turn below.

Commonality

Let us suppose that we want a student to undertake an exploration of a particular mathematical situation taken from Wiliam (1997). The mathematical situation we have in mind is as follows.

> On a square lattice (ie on square centimetre dotty paper), for a given rectangular area, what is the minimum number of lattice points we must mark with a cross so that no lattice point is more than 1 unit from a cross?

This is the core mathematical task *intended* by the teacher (Bauersfeld, 1979). Now we might phrase the problem exactly in this way, and give it to students. In this case, there will be some students who can understand what the problem is asking and get to work straight away. However, there will be others, who would be able to engage in the activity, but for the fact that they don t understand it! For some of these students, the problem will become more accessible if it is located in a familiar context. For example:

> This problem concerns a strange chess piece - a disabled rook - moving over a rectangular chessboard. It can move in the same way as a rook, (ie left and right, forwards and backwards, but not diagonally) except that it can only move one square in each direction. For a given rectangular chessboard, how many rooks do you need to make sure that each square of the board is attacked (or occupied!).

The mathematical problem at the core of this task is the same as before, but it is presented in a metaphorical form. For students familiar with chess, this presentation will be more accessible. However, if the student has no knowledge of chess, this implementation is attempting to capitalise on scripts that the student doesn’t have, and is in fact is likely to hinder more than help. In other words, the metaphor being used is *not sufficiently commonly shared* to be useful. Other variations may however, be more successful.

> Imagine a city whose streets form a square grid, the side of each square being l00m. A policeman stands at a street-corner. He can spot a suspicious person at l00m, so he can watch 400m of street. A single block needs 2 policemen to watch it. 2 blocks will need 3 policemen. What about 3 blocks in a row? 4 blocks in a row? and so on. . .(SMILE, 1990)

This implementation gives a task that is more accessible in that the scripts necessary to engage with this formulation are more widely shared. A simple classification of the extent to which such task metaphors are shared by the students for whom the task or text is intended is as follows:

The task metaphor itself can be classified as:

- universally shared
- commonly shared
- not commonly shared

Unfortunately, in this instance the metaphor is not a good one. There are interpretations consistent with the real-world situation depicted that are not isomorphic to the intended activity. For many students, the idea that someone can see exactly 100 metres, but not 110 metres is plainly absurd. Accordingly, they produce optimal arrangements that accord more with their common sense than with the teacher’s intention, and, as discussed above, this kind of response appears to be more prevalent in female than male students.

Match

The fit between the core mathematical activity intended and the possible interpretations within the ‘policeman’ task is not good. Even ignoring the sexist nature of the task, there are many features in the task description which students are expected to suppress. The expectation made of students is similar to those made by Dienes when he proposed the use of different sized ‘base apparatus’ for the teaching of place value. It was intended that students should abstract the important features (that within a base system the ratio between to different sized pieces is a power of the radix) while other features (such as being made of plastic) would be suppressed.

In general, certain attributes of the task metaphor will map onto corresponding attributes within the target activity, while others will not. The salience of the undesired features contributes crucially to the ‘difficulty’ of the task, since many students will be misled into tackling different kinds of activity.

For other students, the following implementation may lead to greater convergence on the intended activity.

> Imagine a city whose streets form a square grid, the size of each square being l00m. The Fire Department has hoses that are 100 metres long, so a single block needs 2 hydrants to protect

> it. 2 blocks will need 3 hydrants. What about 3 blocks in a row? 4 blocks in a row? and so on. Investigate further. (Graded Assessment in Mathematics, 1988)

The degrees of match between the task metaphor and the intended core activity can be classified as:

- a perfect match
- different in only insignificant details
- different is significant details
- no match

Range

A third feature of models or metaphors for mathematics teaching that needs to be considered is the range that such a model may have—in other words, how far does it take you? Envisaging negative numbers as temperatures above or below zero may be very useful for understanding ordering of integers, and may even provide good pictures for adding directed number. However, contriving plausible scenarios where temperatures below zero are subtracted from each other, or are multiplied is rather more difficult! In contrast, when multiplication of directed numbers is represented as ‘gearboxes’, with negative numbers involving a change of direction, multiplication of directed numbers is modelled quite straightforwardly, but there is no easy way to represent addition and subtraction.

A particular model or metaphor will suit some aspects of a topic better than others. Fractions as ‘lots of’ rectangular cakes can take you all the way from what a fraction through equivalent fractions, to multiplying vulgar fractions. Addition of fractions, however, cannot be accommodated within this model.

If good concept formation comes from identifying commonalities of structure between different models, then the range of a particular model may not be particularly important: the more the merrier. However, a proliferation of models may serve to reinforce the idea that, for example, “negative numbers are temperatures below zero, but when you subtract them, they are shifts on the number line, and when you divide them, they are gradients of lines”.

6 Conclusion

In this paper, a range of examples of the use of ‘real world’ contexts in mathematics has been presented. At one extreme, tasks like ‘Matrix Soccer’ use contexts that are selected entirely for their motivational appeal. The relevance of such tasks is just like the MacGuffins in Hitchcock’s films; a device for motivating the action, to which relatively little attention is paid, and which is frequently introduced only at the last minute. In the example of Alan and his beer from Harper there is a weak structural relationship between the context and the intended mathematics, but this is trivially easy compared to the difficulty of the the number operation required. However ‘bogus’ this context might be, it does not appear to make the problem any harder than the decontextualised example, which cannot be said for Lawson’s disc-jockey example. Here, the context is worse than bogus. Here, thinking your way into the problem—trying to solve the problem as if it were a ‘real world’ problem—makes it much harder, and leads one away from the answer wanted by the teacher.

The ‘realistic’ lift example contains almost as many defects. Admittedly, the structural affinities between the ‘real world’ situation and students’ strategies mark some improvement over the other examples, but the problem is in no way realistic. Thinking your way into the problem, making it your own, will actually give you a harder problem to solve (what is the relationship between the time people are prepared to wait and the number of floors they need to ascend), but one your teacher is likely to regard as wrong.

There, therefore, an inherent and unavoidable tension in the use of contexts in teaching and assessing mathematics. If the teacher claims that a problem is ‘real’, then a student may come to ‘own’ the problem, but may well produce resolutions of the problem that are perfectly acceptable to the student, but involve no mathematics. Of course, this rarely happens,

because, by virtue of their lengthy enculturation into the practices of mathematics teachers, most students know that the activity in which they are involved is a form of 'glass bead game', in which knowledge of the world outside the mathematics classroom is never needed, is hardly ever even useful, and as often as not, will lead one in the wrong direction.

If the teacher has in mind to develop particular elements of mathematical subject matter, the choice of contexts must be driven by the extent to which there is a good match between intended mathematical activity and the 'real world' context, that the relevant aspects of the real world contexts are shared by students, and that the context allows for development of the mathematical subject matter.

7 References

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Address for correspondence: Cornwall House, Waterloo Road, London SE1 8WA. Telephone: 0171 972 3153; Fax: 0171 872 3182; Email: dylan.wiliam@kcl.ac.uk